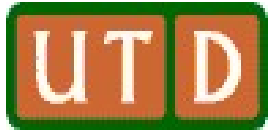




Functional Dependency

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Slides are taken from Silberschatz et al.



Functional Dependencies

- Let R be a relation schema

$$\alpha \subseteq R \text{ and } \beta \subseteq R$$

- The **functional dependency**

$$\alpha \rightarrow \beta$$

holds on R if and only if for any legal relations $r(R)$, whenever any two tuples t_1 and t_2 of r agree on the attributes α , they also agree on the attributes β . **That is,**

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$



Example

- **Example:** Consider $r(A,B)$ with the following instance of r .

A	B
1	3
1	6
2	7

- **On this instance**, $A \rightarrow B$ does **NOT** hold, but $B \rightarrow A$ does hold.



Example

A	B	C	D
a1	b1	c1	d1
a1	b1	c1	d2
a1	b2	c2	d1
a2	b1	c3	d1

- Does $AB \rightarrow C$ hold?
- Does $ABC \rightarrow D$ hold ?
- Does $BC \rightarrow D$ hold?



Example

SSN	LastName	FirstName	City
111111111	Smith	John	Richardson
222222222	Li	Peng	Richardson
333333333	Kant	John	Plano
444444444	Smith	Mark	Plano

- Does $\{ssn\} \rightarrow \{LastName\}$ hold?
- Does $\{ssn\} \rightarrow \{LastName, FirstName\}$ hold ?
- Does $\{LastName, FirstName\} \rightarrow \{City\}$ hold?
- Does $\{City\} \rightarrow \{FirstName\}$ hold?



Procedure for Computing F^+

$F^+ = F$

repeat

for each functional dependency f in F^+

 apply reflexivity and augmentation rules on f

 add the resulting functional dependencies to F^+

for each pair of functional dependencies f_1 and f_2 in F^+

if f_1 and f_2 can be combined using transitivity **then**

 add the resulting functional dependency to F^+

until F^+ does not change any further



Example

-
- $R = (A, B, C, G, H, I)$
 $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$
 - some members of F^+
 - $A \rightarrow H$
 - by transitivity from $A \rightarrow B$ and $B \rightarrow H$
 - $AG \rightarrow I$
 - by augmenting $A \rightarrow C$ with G , to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
 - $CG \rightarrow HI$
 - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$, and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$,
and then transitivity



Closure of Attribute Sets

- Given a set of **attributes** α , define the *closure* of α under F (denoted by α^+) as the set of attributes that are functionally determined by α under F
- Algorithm to compute α^+ , the closure of α under F
result := α ;
while (changes to *result*) **do**
 for each $\beta \rightarrow \gamma$ **in** F **do**
 begin
 if $\beta \subseteq \textit{result}$ **then** *result* := *result* $\cup \gamma$
 end



Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H\}$
- $(AG)^+$
 1. $result = AG$
 2. $result = ABCG$ ($A \rightarrow C$ and $A \rightarrow B$)
 3. $result = ABCGH$ ($CG \rightarrow H$ and $CG \subseteq AGBC$)
 4. $result = ABCGHI$ ($CG \rightarrow I$ and $CG \subseteq AGBCH$)
- Is AG a candidate key?
 1. Is AG a super key?
 1. Does $AG \rightarrow R$? == Is $(AG)^+ \supseteq R$
 2. Is any subset of AG a superkey?
 1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$
 2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$



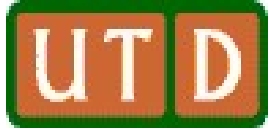
Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
 - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
 - Parts of a functional dependency may be redundant
 - E.g.: on RHS: $\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$ can be simplified to
$$\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$$
 - E.g.: on LHS: $\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$ can be simplified to
$$\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$$
- Intuitively, a canonical cover of F is a “minimal” set of functional dependencies equivalent to F , having no redundant dependencies or redundant parts of dependencies



Extraneous Attributes

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F .
 - Attribute A is **extraneous** in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
 - Attribute A is **extraneous** in β if $A \in \beta$ and the set of functional dependencies $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F .
- *Note:* implication in the opposite direction is trivial in each of the cases above, since a “stronger” functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
 - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (i.e. the result of dropping B from $AB \rightarrow C$).
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
 - C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C



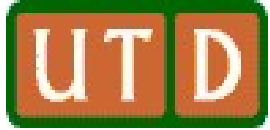
Testing if an Attribute is Extraneous

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F .
- To test if attribute $A \in \alpha$ is extraneous in α
 1. compute $(\{\alpha\} - A)^+$ using the dependencies in F
 2. check that $(\{\alpha\} - A)^+$ contains β ; if it does, A is extraneous in α
- To test if attribute $A \in \beta$ is extraneous in β
 1. compute α^+ using only the dependencies in $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$,
 2. check that α^+ contains A ; if it does, A is extraneous in β



Canonical (Minimal) Cover

- A **canonical or minimal cover** for F is a set of dependencies F_c such that
 - F logically implies all dependencies in F_c , and
 - F_c logically implies all dependencies in F , and
 - No functional dependency in F_c contains an extraneous attribute, and
 - Each left side of functional dependency in F_c is unique.
- To compute a canonical cover for F :
 - repeat**
 - Use the union rule to replace any dependencies in F
 $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$
 - Find a functional dependency $\alpha \rightarrow \beta$ with an
extraneous attribute either in α or in β
/* Note: test for extraneous attributes done using F_c , not F */
 - If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$
 - until** F does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied



Dependency Preservation

- Let F_i be the set of dependencies F^+ that include only attributes in R_i .
 - A decomposition is **dependency preserving**, if
$$(F_1 \cup F_2 \cup \dots \cup F_n)^+ = F^+$$
 - If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive.



Testing for Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of R into R_1, R_2, \dots, R_n we apply the following test (with attribute closure done with respect to F)
 - $result = \alpha$
 - while** (changes to $result$) **do**
 - for each** R_i in the decomposition
 - $t = (result \cap R_i)^+ \cap R_i$
 - $result = result \cup t$
 - If $result$ contains all attributes in β , then the functional dependency $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \dots \cup F_n)^+$



Example

- $R = (A, B, C)$
 $F = \{A \rightarrow B$
 $B \rightarrow C\}$
Key = $\{A\}$
- R is not in BCNF
- Decomposition $R_1 = (A, B)$, $R_2 = (B, C)$
 - R_1 and R_2 in BCNF
 - Lossless-join decomposition
 - Dependency preserving