Functional Dependency

Murat Kantarcioglu

Functional Dependencies

- Let *R* be a relation schema

$$\alpha \subseteq R \text{ and } \beta \subseteq R$$

- The functional dependency

$$\alpha \rightarrow \beta$$

holds on *R* if and only if for any legal relations *r*(R), whenever any two tuples t_1 and t_2 of *r* agree on the attributes α, they also agree on the attributes β. That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

Example

- Example: Consider *r*(A,*B*) with the following instance of *r.*

A	B
1	3
1	6
2	7

- On this instance, $A \rightarrow B$ does **NOT** hold, but $B \rightarrow A$ does hold.

Example

A	B	C	D
a1	b1	c1	d1
a1	b1	c1	d2
a1	b2	c2	d1
a2	b1	c3	d1

- Does AB $\rightarrow$ C hold?
- Does ABC $\rightarrow$ D hold ?
- Does BC $\rightarrow$ D hold?

Example

SSN	LastName	FirstName	City
111111111	Smith	John	Richardson
222222222	Li	Peng	Richardson
333333333	Kant	John	Plano
444444444	Smith	Mark	Plano

- Does {ssn} $\rightarrow$ {LastName} hold?
- Does {ssn} $\rightarrow$ {LastName,FirstName} hold ?
- Does {LastName, FirstName}$\rightarrow$ {City} hold?
- Does {City} $\rightarrow$ {FirstName} hold?

Procedure for Computing F^+

$F^+ = F$

repeat

for each functional dependency f in F^+
apply reflexivity and augmentation rules on f
add the resulting functional dependencies to F^+

for each pair of functional dependencies f_1 and f_2 in F^+
if f_1 and f_2 can be combined using transitivity **then**
add the resulting functional dependency to F^+

until F^+ does not change any further

Example

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H\}$
- some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$, and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$, and then transitivity

Closure of Attribute Sets

- Given a set of attributes α, define the *closure* of α under *F* (denoted by α^+) as the set of attributes that are functionally determined by α under *F*
- Algorithm to compute α^+, the closure of α under *F*

```
result := α;
while (changes to result) do
    for each β → γ in F do
      begin
        if β ⊆ result then  result := result ∪ γ
      end
```

Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H\}$
- $(AG)^+$
  1. $result = AG$
  2. $result = ABCG$ $(A \rightarrow C$ and $A \rightarrow B)$
  3. $result = ABCGH$ $(CG \rightarrow H$ and $CG \subseteq AGBC)$
  4. $result = ABCGHI$ $(CG \rightarrow I$ and $CG \subseteq AGBCH)$
- Is AG a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R$? == Is $(AG)^+ \supseteq R$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$
     2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$